Examination "Toepassingen van de Quantum-Fysica" Groningen, 17-11-98

Please each problem on a separate sheet of paper (to facilitate correction by different people). Please put your name on each sheet!

Problem 1.

Consider a He atom in the groundstate

a) the binding energy of both electrons together is 78.9 eV. Determine the effective nuclear charge "seen" by each of the electrons. Use the fact that the binding energy of the 1s-electron in hydrogen is 13.6 eV
b) what is the minimum energy, necessary to singly ionize the He atom?
c) consider now the case that both electron are in the n=2 shell: what are the configurations of the possible 2-electron states
d) which of the configurations is most strongly bound? Give arguments voor your answer!
e) give the possible terms and states for each of the configurations;
f) in a schematic drawing give the relative positions of all states for which the *l*-quantum nunbers of both electrons are identical

Problem 2. Magnetic Resonance Imging (MRI) of the human lung

The human lung can be visualized via MRI when the lung is filled with ^{3}He and nuclear magnetic resonance transitions are observed, which are induced by radiation of suitable frequencies. Let us consider the physics of this in some more detail: the nucleus of ^{3}He consists of two protons and one neutron, has a spin of I=1/2 and a magnetic moment of $\mu = -2.1\ \mu_n$. We first consider a ^{3}He atom in its electronic ground state He($1s^2$)^{1}S;

a) what is the total magnetic moment of the atom (in multiples of the Bohr magneton μ_B)?
b) in a magnetic field B the groundstate is split due to the Zeeman effect of the hyperfine states. Which frequency is required to induce transitions between these states in a field of B = 1 Tesla?
c) What is the difference in population (in %) between the relevant states at room temperature (T=300 K)

In order to reach a stronger population difference one can use circularly polarized laser light which induces transitions He($1s2s$)^{3}S → He($1s2p$)^{3}P, whereby the He($1s2s$)^{3}S atoms are in the first place obtained by electron collisions in a discharge. In order to find the most suitable states of the ^{3}S and the ^{3}P term we look which states are possible:

d) what are the possible values of the total electronic angular momentum **J=L+S** for various states of the ^{3}S and the ^{3}P terms? (bold face indicates that the quantities are vectors!)
e) what are the possible total angular momenta **F=J+I** of the corresponding hyperfine states?
f) What is the most suitable hyperfine transition which should be induced in order to obtain nuclear spin orientation by irradiation with circularly polarized laser light? Make a schematic drawing in which also the magnetic substates are shown and indicate which transitions - induced or spontaneous - will take place

$\mu_B = 9.3 \cdot 10^{-24}$ Am^2 (=J/T) $\quad$ $h = 6.6 \cdot 10^{-34}$ Js $\quad$ Boltzman constant $k = 1.38 \cdot 10^{-23}$ J/K

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Problem 3.

A monochromatic beam of atoms (i.e. all atoms have the same velocity V_0) is decelerated by a collinear laser beam propagating in opposite direction. For simplicity we assume a pure two-level atom with enegy levels E_1 and E_2 (with $E_1 < E_2$).

a) Explain qualitatively why the atoms can be decelerated in this way; indicate in a schematic drawing which induced and spontaneous transitions may play a role.

b) Show that the average momentum change of the atoms is given by

$$\frac{dp}{dt} = -\frac{h}{\lambda}\frac{N_1 - N_2}{N} B\rho$$

with N_1 and N_2 the population of the two states involved, N the total nunber of atoms, B the relevant Einstein coëfficient for stimulated absorption and emission and ρ the spectral energy density of the laser light.

c) Use the above relation to find a relation between *dp/dt,* the Einstein *A* coëfficient for spontaneous transitions and the fraction $f=N_2/N$ of the excited atoms, whereby it is assumed that there is a stationary population distribution between N_1 and N_2.

d) Due to the deceleration the velocity of the atoms changes as a function of time. To keep the laser in resonance the laserfrequency ν_L has therefore to be adjusted in time. Show that the time dependent variation (the "chirp") of ν_L is described by the differential equation

$$\frac{d\nu_L(t)}{dt} = -\frac{h\nu_r}{mc^2} Af\nu_L(t)$$

whereby ν_r is the resonance frequency with the atoms at rest.

e) How does the laserfrequentie have to be changed as a function of time in order keep the light resonant with the decelerated atoms?

Problem 4. Perturbation theory

We consider a one-dimensional potential well with infinitely high walls and a flat bottom given by V=0 for $|x|<a$ and $V=\infty$ for $|x|>a$.

a) give the wavefunctions u_0 and u_1 for the two lowest eigenstates of a particle in this well.

b) Calculate the energy eigenvalues E_0 and E_1 for these two states.

Now we consider a perturbation of the potential given by $V_p=-A \sin(\pi x/2a)$ for $|x|<a$, with $A<<E_0$.

c) give the Schrödinger equation for this situation

d) why will first order perturbation calculation in this situation not give a satisfactory result?

e) In order to find the modified eigenfunctions and energy eigenvalues we introduce new eigenfunctions consisting of a linear combination of u_0 and u_1, i.e. $\Psi = bu_0+cu_1$. Describe qualitatively how you can calculate the new eigenenergies and eigenfunctions.

f) Caculate the new eigenenergies. Make use of the enclosed integral table. Give an approximate expression by exploiting that $A<<E_1-E_0$.

g) Calculate the new eigenfunctions.